

# Loudspeakers in Vented Boxes: Part II\*

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## VIII. LOUDSPEAKER EFFICIENCY

In Eq. (12) an expression was derived for the efficiency of a loudspeaker in a box, which consists of three parts. We have considered, in the meantime, the third part which varies with frequency. We now consider the first two parts. Thus the basic efficiency

$$\eta_{ob} = (\rho_0/4\pi c)(B^2 l^2 S_d^2 / R_e M_{ms}^2). \quad (66)$$

If this experience is compared with Beranek's Eq. (7.19) it will be seen to give one quarter of his value, after the differences in notation are allowed for.

1) Multiplication by 100 to give percentage.

2) The definition of "nominal input power" in Eq. (10) of this paper as the power delivered by the amplifier into the nominal speaker impedance  $R_e$ <sup>3</sup>. Beranek's treatment is based on the idea of maximum power transfer when the load impedance is equal to the generator impedance, as in his Eq. (7.14). If this condition,  $R_g = R_e$ , is substituted in his Eq. (7.19), one of the conditions for agreement with Eq. (66) is satisfied. However, in dealing with the output power from an amplifier, the writer prefers to consider the power delivered into the load without regard to the output impedance  $R_g$ , for the

relationship of  $R_g$  to the optimum load impedance depends in the first place on the nature of the output device, transistor, pentode, or triode. Furthermore,  $R_g$  can be manipulated by feedback techniques (see Section XII) to almost any desired value without affecting the condition for optimum output power. Hence the treatment in this paper.

3) The lumping in this paper of all mechanical mass into  $M_{ms}$ .

The additional multiplication factor of one quarter arises from the following.

4) Beranek's figure being for the radiation from *both* sides of the diaphragm, giving twice the output from one side.

5) The assumption in this paper that the radiation resistance in a box is that of a piston at the end of a long tube [3, p. 216]. This radiation resistance is one half of that of a piston in an infinite baffle.

Thus the results are consistent. We will continue here to use  $\eta_{ob}$ , unless stated otherwise. But it is important to define efficiency in terms of actual use and to remember that the value of  $\eta_{ob}$ , being the basic efficiency in a box, is one half the efficiency on an infinite baffle and one quarter of the efficiency, if radiation from both the front and back of a speaker in an infinite baffle is considered.

To simplify the understanding of Eq. (66), we make a further substitution. It can be shown that

$$l^2/R_e = V_{cu}/2\sigma \quad (67)$$

where  $\sigma$  is the resistivity of the conductor and  $V_{cu}$  is the volume of the conductor assumed to be completely within the air gap. In so far as the conductor overlaps the air gap a correction factor would be applied. Then

<sup>3</sup> The nominal impedance of a loudspeaker is usually taken as the minimum impedance at mid-frequencies, at  $f_n$  in Fig. 5. This is a little greater than  $R_e$ ; but for simplicity, and it is hoped without too much confusion, the nominal impedance is taken here as  $R_e$ .

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Eq. (66) becomes

$$\eta_{ob} = (\rho_0/8\pi c\sigma) (B^2 S_d^2 V_{cu}/M_{ms}^2) \quad (68)$$

that is, once the voice coil conductor material, and therefore  $\sigma$ , is chosen, the loudspeaker efficiency depends on the four parameters in the second bracket. Without digressing too far into the problem of loudspeaker design, it is noted that this shows the two basic questions in loudspeaker design for good efficiency at low frequencies.

1) How to make the product  $B^2 V_{cu}$  a maximum for a given magnet, since the larger  $V_{cu}$  is made, the wider and/or deeper is the air gap, and hence the lower is  $B$ .

2) How to make  $S_d^2/M_{ms}^2$  a maximum, since the larger the area the greater the mass for a given cone thickness. If thickness is reduced, break-up problems increase due to nonlinearity of the piston drive. In conventional designs the mass of the voice coil is small (less than 20%) compared with the mass of the cone, so there is little interaction between  $V_{cu}$  and  $M_{ms}$ .

The writer prefers to express efficiency as an electro-acoustic conversion loss

$$dB_{ea} = 10 \log_{10} \eta. \quad (69)$$

For example, 1% efficiency is equivalent to 20-dB electro-acoustic conversion loss. This facilitates comparisons between different designs and estimations of the acoustic level (in phons) which a speaker will provide with a given amplifier and listening room (see Appendix).

## IX. RELATIONSHIP OF EFFICIENCY $\eta$ , $Q$ , AND BOX VOLUME

First we take Eq. (57) and break  $Q_t$  into two component parts, one due to the acoustic resistances and the other due to electrical damping, so that

$$1/Q_t = 1/Q_a + (1/Q_r) [R_c/(R_g + R_c)]. \quad (70)$$

Then from Eqs. (8) and (57), the acoustic  $Q$  of the loudspeaker

$$Q_a = \omega_s M_{as}/R_{as} \quad (71)$$

and the electrical  $Q$  of the loudspeaker

$$Q_r = \omega_s M_{as} R_c S_d^2 / B^2 l^2 \quad (72)$$

i.e.,

$$Q_r = 2\sigma \omega_s M_{ms} / B^2 V_{cu}. \quad (73)$$

Again if we consider the approximate relationship established in Table I that

$$C_{as} f_s^2 / C_{ab} f_3^2 \cong \sqrt{2} \quad (74)$$

thus, converting the acoustic compliance of the box into the equivalent volume of air, the box volume

$$V_b \cong (\rho_0 c^2 / \omega_3^2 \sqrt{2}) (S_d^2 / M_{ms}) \quad (75)$$

remembering that this approximate relationship holds only in the absence of amplifier assistance.

Now considering together Eqs. (68), (73), and (75), the following points emerge.

1) The same considerations that ensure high efficiency also ensure a low  $Q_r$ , except that  $Q_r$  is independent of the projected piston area  $S_d$  and depends only on the first power of the cone mass  $M_{ms}$  instead of the second power.

2) The box volume depends, apart from the choice of

cutoff frequency  $f_3$ , only on  $S_d^2$  and  $M_{ms}$ . Reduction of box volume by reduction of  $S_d$  involves an increased cone excursion, which is inversely proportional to  $S_d$  and  $\omega_b^2$ , for a given acoustic power. If the box volume is reduced by increasing  $M_{ms}$ ,  $\eta$  is decreased even more (see Eq. (68)), necessitating increased amplifier power. It would seem that the well-known R-J enclosure works this way. The opening in front of the cone is restricted, and this increases the air mass loading  $M_{a1}$  of Fig. 1 in the same manner as a vent. Thus  $M_{ms}$  is increased and the box volume  $V_b$ , i.e.,  $C_{ab}$ , for a given low-frequency cutoff is reduced, but at the price of reduced efficiency throughout the piston range.

3) The best way of increasing  $\eta$  and lowering  $Q_c$  is to increase the flux density  $B$ . But if one starts with a reasonably high value of  $B$  in the first place, the cost of obtaining an extra decibel of efficiency increases rapidly. So again to obtain a given amount of acoustic power at a given price, a compromise must be struck between the sizes of magnet, box, and amplifier. However, this discussion does show the reason for the large magnet, long throw, heavy cone designs used overseas in small "book-shelf boxes."

Note that  $Q_a$  in Eq. (71) depends only on acoustic reactance and resistance, that is,  $Q_a$  is independent of  $B$ .

Substituting Eqs. (58) and (73) in (68), we obtain the interesting relationship

$$\eta_{ob} = \omega_s^3 V_{as} / 4\pi c^3 Q_c \quad (76)$$

where  $V_{as}$  is the volume of air equivalent to the acoustic compliance of the loudspeaker, or

$$\eta_{ob} = 8.0 \times 10^{-12} f_s^3 V_{as} / Q_c \quad (77)$$

where  $V_{as}$  is in cubic inches. Thus the basic efficiency of the speaker can be calculated from the three parameters which are used for the design of the box. A physical explanation of the variation of  $\eta$  and  $Q_c$  is given at the end of Section XII.

## X. EXCURSION OF LOUDSPEAKER CONE

In the derivation of Eq. (12) it was found that

$$\begin{aligned} U_c / (U_c - U_p) &= 1 - 1/\omega^2 M_{ar} C_{ab} \\ &= 1 - (\omega_b/\omega)^2. \end{aligned} \quad (78)$$

Thus the acoustic output power radiated by the cone alone is

$$W_{aoc} = W_{ei} \eta_{ob} [1 - (\omega_b/\omega)^2]^2 |E(j\omega)|^2. \quad (79)$$

Now starting from the relationship

$$W_{aoc} = (R_{ma} \dot{x}^2) 10^{-7} \quad (80)$$

which is [4, Eq. 6.13], where  $R_{ma}$  is the mechanical radiation resistance and  $\dot{x}$  is the rms velocity of the piston in cm/s, it is possible to derive an expression for peak cone movement,

$$x_{pk} = 1.31 \times 10^5 \sqrt{W_{aoc}} / f^2 S_d \quad (81)$$

or

$$x_{pk} = 5.17 \times 10^6 \sqrt{W_{aoc}} / \omega^2 S_d \quad (82)$$

where  $x_{pk}$  is in inches (note that this  $x$  which stands for excursion is unrelated to the shape parameter  $x$  of Eq.

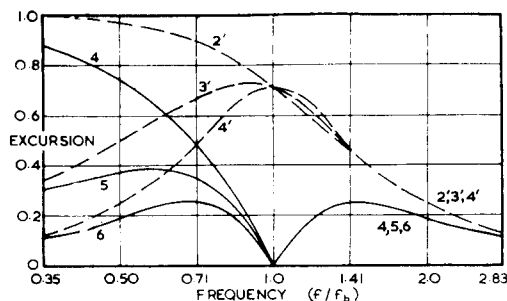


Fig. 10. Normalized cone excursion versus normalized frequency for various orders of Butterworth response with loudspeaker in vented box (solid curves) and in infinite Baffle (dashed curves). Curves are numbered for order of response. Normalized excursion is  $|(f_b/f)^2 - (f_b/f)^4| \cdot |E(j\omega)|$ , part of Eq. (84).

(21) *et seq.*),  $S_d$  is in square inches, and  $W_{avc}$  is in watts. Again allowance is made for the fact that the loudspeaker is mounted in a box so that the radiation resistance is half the value for an infinite baffle. Thus Eqs. (81) and (82) will give values for displacements which are  $\sqrt{2}$  times those given in [4, Fig. 6.9]. Thus

$$x_{pk} = 5.17 \times 10^6 (\eta_{oh} W_{av})^{1/2} [1 - (\omega_b/\omega)^2] |E(j\omega)| / \omega^2 S_d \quad (83)$$

If we write this expression as

$$x_{pk} = [1.31 \times 10^5 (\eta_{oh} W_{av})^{1/2} / f_b^2 S_d] [(f_b/f)^2 - (f_b/f)^4] |E(j\omega)| \quad (84)$$

it is apparent that there are two parts, one fixed for a given speaker and box (note frequency  $f_b$  in this expression) and one that varies with frequency. This latter expression is plotted in Fig. 10 for various Butterworth responses, in which box, speaker, and cutoff frequencies are identical. The solid curve 4 gives the excursion of the classical fourth-order Butterworth alignment no. 5 of Table I. Solid curve 5 refers to the fifth-order Butterworth alignment no. 10, which includes a simple auxiliary filter. Solid curve 6 refers to the sixth-order Butterworth alignment which is identical for nos. 15, 20, and 26, since both frequency response and box resonant frequency are the same in each. For comparison, the dotted curves give the excursions for the same speaker in an infinite baffle (totally enclosed box) with the same power. Dotted curve 2 applies to a speaker with a second-order Butterworth response ( $Q_t = 0.707$ ). Dotted curve 3 applies to a third-order Butterworth response ( $Q_t = 1$ , with a simple auxiliary filter). Dotted curve 4 applies to a fourth-order Butterworth response ( $Q_t = 1.307$ , with a second-order auxiliary filter). The frequency response is the same as solid curve 4, but it is obtained by different means. The curves show the following.

1) The excursion below resonance is reduced greatly in both vented box and infinite baffle when an auxiliary highpass filter is used. The first-order auxiliary filter gives a good improvement especially in view of its simplicity. The second-order auxiliary filter not only allows a greater reduction of cone excursion, it also allows the use of three separate box alignments for the same response and allows box volume to be traded for amplifier power in the case of the vented box. The Butterworth curves with second-order auxiliary filters are symmetrical about the

center frequency. There seems little need therefore to use more elaborate filtering.

2) Even more important, the excursion of the cone is reduced greatly when the loudspeaker is placed in a vented box. The curve predicts zero excursion at the box frequency. This arises from the assumption that the  $Q$  of the box circuit is infinite. While this cannot be achieved completely in practice, the excursion at the box frequency will be low so long as the ratio of  $Q$  of the box to  $Q$  of the speaker is high, as demonstrated in Section II.

Of course, if resistance is deliberately introduced into the box circuit, as by making the vent from a number of small holes or by stretching fabric across the vent, the  $Q$  will be greatly reduced and some of the advantage of the vented box will be lost, as shown in the next section. Fig. 10 refers only to Butterworth responses. In Fig. 11, a plot is made of the function  $[(\omega_b/\omega)^2 - (\omega_b/\omega)^4]$  against frequency. If, for example, in a Chebyshev response the frequency response is known, the excursion at different frequencies can be found by reading off the function at a given frequency on Fig. 11 and multiplying it with the frequency response. The rapid rise of the function between normalized frequencies of 1 and 0.71 shows why responses should be preferred in which  $f_b$  is not too much greater than  $f_3$ . Thus with respect to cone excursion, an alignment in the group 20-25 would be preferred to its counterpart in the group 15-19 which has a lower value of  $f_3/f_b$ .

It would seem that in published ratings of loudspeakers, the maximum excursion  $x_{max}$  would be more useful than the conventional rating of maximum input power. The latter might save the loudspeaker from a melted voice coil, but when mechanical damage or undistorted acoustic output are of interest,  $x_{max}$ , along with the kind of baffle and the alignment, determine the performance.

## XI. BOXES WITH RESISTIVE LOADING OF VENT

Good results have been reported with resistively loaded vents [1]. These were therefore investigated using both series and parallel loading of the vent as shown in Fig. 12. In both cases, the resistance was assumed to be constant with respect to frequency and the response function was found to be of third order.

This, by the way, explains a discrepancy between the statements in [3, p. 244] and in [2, p. 11] that the drop in response below cutoff is 18 dB per octave, even though [2, Eq. 15)], which is equivalent to Eq. (20) of this

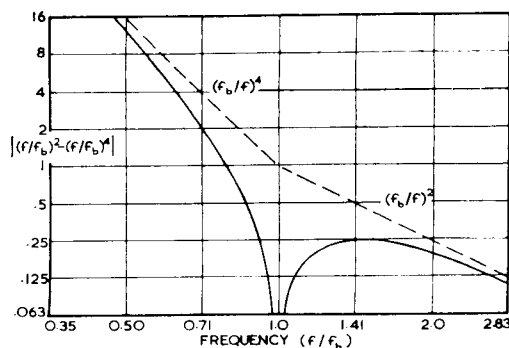


Fig. 11. Function  $|(f_b/f)^2 - (f_b/f)^4|$  versus normalized frequency  $f/f_b$ . The function, part of Eq. (84), is used to compute excursion when frequency response  $|E(j\omega)|$  is known.

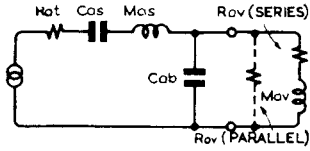


Fig. 12. Equivalent acoustic circuit of loudspeaker and box showing added acoustic damping in series or parallel with vent.

paper, obviously has an asymptotic slope of 24 dB per octave. In the practical case, where resistance loading of the vent however small will be encountered, the asymptotic slope will eventually be 18 dB per octave; but so long as the original simplifying assumptions hold, the response in the region that concerns us will be effectively 24 dB per octave.

The expressions are, for the case of series resistance loading,

$$E(p) = 1/\{1 + (1/p)(1/Q_b T_b + Q_b T_b/T_s^2) + (1/p^2)(1/T_s^2 + 1/T_b^2 + C_{as}/C_{ab} T_s^2) + Q_b/p^3 T_s^2 T_b\} \quad (85)$$

when

$$1/Q_t = T_s/Q_b T_b + Q_b T_b/T_s \quad (86)$$

and  $Q_b$  is defined as the ratio of acoustic mass resistance to series acoustic resistance of the vent at the box resonant frequency.

For the case of parallel resistance loading,

$$E(p) = 1/\{1 + (1/p)(Q_b T_b/T_s^2 + 1/Q_b T_b + C_{as} T_b/C_{ab} T_s^2 Q_b) + (1/p^2)(1/T_s^2 + 1/T_b^2 + C_{as}/C_{ab} T_s^2) + Q_b/p^3 T_s^2 T_b\} \quad (87)$$

when

$$1/Q_t = Q_b T_b/T_s + T_s/Q_b T_b + C_{as} T_b/C_{ab} T_s Q_b \quad (88)$$

and  $Q_b$  in this case is the ratio of parallel acoustic resistance across the vent (series resistance being assumed negligible) to acoustic mass reactance. Note the inversion of the expression for parallel  $Q_b$  compared with that for series  $Q_b$ . Since these equations are of third order and there is one extra variable  $Q_b$ , there are two extra degrees of freedom in the design. However, one is removed if an all-pole function is desired, hence Eqs. (86) and (88). Before an alignment is commenced, one other parameter must be fixed arbitrarily. The ratio  $C_{as}/C_{ab}$  seems the easiest to handle for this purpose. Thus in a third-order Butterworth alignment, if  $C_{as}/C_{ab}$  is made 1.414, for comparison with the fourth-order Butterworth alignment no. 5 of Table I, the results are as given in Table II.

Table II. Parameters for third-order Butterworth alignment with resistive vented loading.

Method of Loading	$f_3/f_s$	$f_3/f_b$	$C_{as}/C_{ab}$	$Q_t$	$Q_b$
Series Resistance	1.317	1.285	1.414	0.379	2.22
Parallel Resistance	1.420	1.120	1.414	0.352	2.25
No Resistance (Alignment No. 5, for comparison)	1.000	1.000	1.414	0.383	$\infty$

It will be seen that although the box had the same volume, the cutoff frequencies for the resistively loaded alignments are 1.32 and 1.42 times higher than no. 5 of Table I. Compared with previous alignments (no. 1–9 of Table I) those of Table II are most inefficient in utilization of box volume, there is no compensating freedom to use a larger value of  $Q_t$ , in fact it needs to be a little smaller, finally and more important, the excursion of the speaker near cutoff frequency is greatly increased. For these reasons, the use of acoustic damping seems to be unjustified. It is realized that the cases treated here use resistances which are constant with frequency. Some acoustic resistances, as described for example in [3, Eqs. (5.54) and (5.56)], vary with frequency and might have a somewhat different effect. However, the use of added damping with the attendant dissipation of input power seems to be wrong in principle, unless a suitable alternative cannot be found. It is believed that the method outlined already provides the suitable alternative.

### Effect of Losses in Box and Vent

Having established that intentional loading of the vent is undesirable, it is of interest to know the effect on the ideal response, obtained by assuming zero loss, of small unavoidable losses in the box and vent. We will only consider performance at the box resonant frequency, since at this frequency 1) the box circuit contributes most, in the ideal case all, of the acoustic output, and 2) the losses in the box circuit are greatest.

In the ideal case, the transfer impedance connecting the input force  $E_g B l / S_d (R_g + R_e)$  with the vent volume velocity  $U_p$  in Fig. 2, at the box resonant frequency  $\omega_b$ , is  $j\omega_b M_{av}$ . If now we express all the losses in the vent and the box as  $Q_b$ , the “ $Q$  of the box and vent circuit,” the transfer impedance, and thus the frequency response at  $\omega_b$  is reduced by a factor which we will call the maximum box loss  $(A_b)_{max}$ . Then, to a close approximation,

$$(A_b)_{max} = 1/[1 + (1/Q_t Q_b)(C_{ab}/C_{as})(\omega_b/\omega_s)]. \quad (89)$$

If we apply the approximations of parts 1) and 2) of Section VI for the “unassisted” alignments no. 1–9 of Table I, Eq. (89) is simplified to

$$(A_b)_{max} = 1/[1 + (1.85/Q_b)(f_b^2/f_s^2)] \quad (90)$$

that is, for a given value of  $Q_b$ , the box loss increases with higher values of  $f_b/f_s$  and thus, larger box sizes.

To illustrate the effect of box loss, Eq. (89) is applied to various alignments. Taking first the classical alignment, no. 5 of Table I, the maximum box loss is 0.5 dB when  $Q_b$  is 30 and 1.5 dB when  $Q_b$  is 10. Taking other, extreme, alignments when  $Q_b$  is 30, the losses for alignments no. 1, 9, 19, and 25 are 0.3 dB, 0.7 dB, 0.5 dB, and 0.7 dB, respectively. Thus it can be seen that a  $Q_b$  of 30 will have little effect on any alignment. With a  $Q_b$  of 10, the losses are 0.9 dB, 1.9 dB, 1.5 dB, and 2.2 dB, respectively, i.e., when the box  $Q$  is reduced three times, the maximum box loss is increased approximately three times in each case. A method of measuring  $Q_b$  is given at the end of Section XIV and illustrated in the Appendix.

Table III. Change of output impedance  $R_o$  with type of feedback.

	Negative	Positive
Voltage Feedback	$R_o$ Decreases	$R_o$ Increases
Current Feedback	$R_o$ Increases	$R_o$ Decreases

## XII. AMPLIFIER CIRCUITS

### Negative Output Impedance

It is essential to the method that the overall  $Q_t$  of the loudspeaker plus amplifier be properly controlled within  $\pm 10\%$  for  $\pm 1$  dB accuracy of response. As explained in Section V, if  $Q_t$  is twice the optimum value, a 6-dB peak results. Similarly if  $Q_t$  is too small, there will be a dip in the response. Thus it is important that the speaker  $Q_e$  be known, either from information supplied by the manufacturer or by measurement, and that the amplifier output impedance be then adjusted to give the required overall value of  $Q_t$ . It is assumed in the following that the available speaker  $Q_e$  is larger than the required  $Q_t$ . This is the more usual case, especially with lower priced loudspeakers. But if it is smaller, a suitable adjustment can easily be made, for example, by changing the positive current feedback to negative current feedback.

The subject of amplifier output impedance control properly requires another paper, which it is hoped will be presented later. For the present only some general results will be given.

If feedback is applied to an amplifier, not only does its gain change, but its effective output impedance  $R_o$  changes also; not its optimum load impedance which remains unchanged by feedback but the impedance which is seen when looking back into the amplifier output terminals. The effect of applying different kinds of feedback is shown in Table III.

The terms voltage feedback and current feedback refer of course to feedback of a voltage which is proportional to output voltage and output current, respectively. In the latter case, this is usually achieved by placing a small resistor in series with the load, and taking the voltage drop across it for feedback. It will be seen that not only does negative voltage feedback reduce the output impedance  $R_o$ , positive current feedback reduces  $R_o$  also, and to the greater extent that  $R_o$  can be made zero or negative.

Negative output impedance is characteristic of oscillators; one therefore tends to be wary of it as tending to instability. But this can only happen when the positive output impedance presented by the load is less than the negative impedance presented by the amplifier. Now the impedance of a loudspeaker in a box, typified by Fig. 5, can never be less than its dc resistance  $R_e$  of Fig. 4. The only exception is at very high frequencies, where the shunt capacitance of the connecting leads takes effect. But unless the leads are very long and the nominal impedance of the speaker is high, this will not usually take effect within the bandwidth of the amplifier. And in any case, we will want to eliminate the negative impedance characteristic at the higher audio frequencies for reasons that will be discussed later. Thus a negative impedance amplifier can be made completely stable apart from gross

misadjustment, such as connecting a loudspeaker of much lower impedance than the design figure or short-circuiting the output leads.

The method of applying mixed feedback is shown in Fig. 13. It will be seen that if the sense of the voltage developed across the potential divider  $R_3$  and  $R_4$  is negative, then the voltage developed across the current feedback resistor  $R_2$ , usually made less than  $1/10$  the nominal impedance of the speaker to minimize power loss, will be positive. The circuit shows why this method is sometimes described as bridge feedback. Usually the circuit is arranged to be unbalanced at all frequencies so that the net feedback is always negative, but it need not necessarily be so. For example, if no net negative feedback is desired, so that there is no overall gain reduction with nominal load, the bridge will be balanced at nominal load.

Physically, the circuit can be thought of as having a certain amount of feedback with nominal load, in which the negative voltage feedback is partially neutralized by the voltage from the positive current feedback resistor. If the impedance  $Z_1$  is open-circuited, the current feedback from  $R_2$  disappears leaving a greater amount of negative feedback. Thus the output voltage may be less on open circuit than on nominal load. This is the effect we describe as negative output impedance. Its extent, or whether it is seen at all, will depend on the original gain and output impedance of the amplifier and the value of the feedback resistor  $R_2$ . Thus if we have, as in Fig. 4, a loudspeaker resistance  $R_e$ , and make the effective output impedance of the amplifier  $R_o$  equal to, say,  $-0.6R_e$ , the total effective impedance of  $R_o + R_e$  becomes  $+0.4R_e$ . And if the  $Q_e$  of the loudspeaker is 1.0, this will make the overall  $Q_t$  a value of 0.4 by applying a maximum of  $1.0/0.4$  times, i.e., 8.0 dB, extra gain reduction by negative feedback when the impedance of the speaker becomes high, as at  $f_h$  and  $f_l$  of Fig. 5. (Need it be emphasized that this form of damping does not dissipate amplifier output power, except in the small current feedback resistor. It reduces power by feedback at the source.)

This fact necessitates a degree of additional care in the design of negative impedance amplifier. For when the load is open-circuited, the negative feedback rises to the maximum; in this case a gain reduction of 8 dB above the nominal value, and the stability margin will be reduced. The size of the negative impedance will in practice be limited either by this consideration or by the need for a feedback resistor so large that it dissipates an appreciable part of the output power.

An alternative method of control damping uses a feedback winding closely coupled to the voice coil. In this way, feedback can be taken effectively from the junction of  $R_e$  and  $L_e$  in Fig. 4. Simple negative feedback

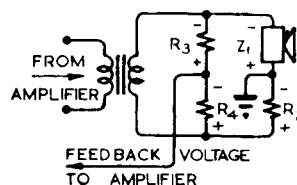


Fig. 13. Method of applying mixed feedback (positive current and negative voltage).

then reduces an effective output impedance which is the sum of  $R_g + R_e$ . Thus  $Q_t$  is reduced in the same way as before. Since the impedance of the feedback circuit is usually high compared with the voice coil impedance, the feedback winding can be made of very fine wire. In fact, if it is wound bifilar with the main winding with wire 16 B&S gauges smaller, it will fit into the air spaces between the larger wires. It thus takes up no more space in the air gap and adds less than 3% to the mass of the copper in the voice coil. Unfortunately, such a winding is difficult to achieve in production and is thus rarely, if ever, used.

If negative impedance is applied, it reduces the output voltage whenever the load impedance is high, i.e., not only in the region of  $f_l$  and  $f_h$  in Fig. 5, but also at frequencies above  $f_n$  where the impedance rise is due to the inductance  $L_c$  of Fig. 4. At high frequencies, this contributes nothing to the acoustic damping of the speaker, but simply reduces the high-frequency response, in the case quoted above, a maximum of 8 dB. This is usually undesirable, so the negative impedance should be eliminated at the higher audio frequencies. One method among several possible is shown in Fig. 14a. Here an inductance  $L_2$  is added to the feedback resistor  $R_2$  with a time constant  $L_2/R_2$  matching that of the speaker, usually in the range of 30–60  $\mu$ s. This can be easily done by winding a solenoid of copper wire which combines resistance  $R_2$  and inductance  $L_2$ . However, since this achieves its result by feeding back an increasing positive voltage to neutralize an increasing negative voltage, quite small unbalance between the two can cause instability at high frequencies.

On the other hand, consider the circuit of Fig. 14b where the lower resistor of the negative feedback potential divider  $R_4$  becomes two resistors  $R_5$  and  $R_6$  in series. Suppose that a suitable set of resistors  $R_2$ ,  $R_3$ , and  $R_4$  has been found to give the correct gain and output impedance for low frequencies with the dotted connection open-circuited. It is then possible to find a tapping point on  $R_4$  (i.e., the junction of  $R_5$  and  $R_6$ ) such that the same gain is obtained on nominal load whether the dotted connection is open circuit or short circuit. This is done by connecting the nominal load and making  $R_5$  and  $R_6$  a potentiometer whose wiper is grounded through a switch. The wiper is adjusted until the gain is the same with the switch open or closed. In the open-circuit condition, the output impedance will be the value originally chosen, but on short circuit, most of the positive current feedback will be eliminated. If then a capacitor is substituted for the switch as shown in Fig. 14b, the output impedance will change from a negative value at low frequencies to a small value, either positive or negative depending on the

particular circuit. The frequency of changeover, which should be, say, two octaves above  $f_h$ , depends on the capacitance  $C$  and the resistances  $R_5$  and  $R_6$ . At the same time, the gain of the amplifier on nominal load stays constant over the whole audio range.

### Auxiliary Filters

The auxiliary filtering needed for sixth-order alignments is best provided by circuits using RC networks in a feedback loop ahead of the main amplifier. In general it is unwise to use the main amplifier feedback loop to provide both negative impedance and high-pass filtering. It is hoped to deal with this in a later paper, but for the moment the reader's attention is directed to the extensive literature, of which [9] and [10] are examples, concerning low-frequency filters without inductors, which use resistors, capacitors, and tubes in comparatively inexpensive combinations.

### Maximum Power at Maximum Impedance

The electrical impedance seen at the terminals of a loudspeaker varies greatly with frequency, but output stages deliver maximum power into a comparatively narrow range of impedances. To consider the maximum acoustic power that can be delivered by an amplifier through a loudspeaker, we return to the equivalent electrical circuit of Fig. 4, together with the impedance curve of Fig. 5. For this purpose, we ignore for the moment the inductance  $L_c$  with its electrical shunt loss  $R_{sh}$  and assume that the curve of Fig. 5 reaches a final value of  $R_c$  above  $f_n$ .

The acoustic output depends on the voltage across  $R_{cs}$ , which includes the electrical equivalent of the radiation resistance  $R_{ar1}$ . Since  $R_{ar1}$  varies with frequency squared, the voltage across  $R_{cs}$  needs to vary inversely with frequency to maintain constant acoustic power. At the higher frequencies the motional impedance is much lower than  $R_c$  and is controlled by the reactance of  $C_{mes}$ , which is equal to  $B^2 l^2 / M_{ms}$ . Thus the condition for flat response is achieved, often described as mass control.<sup>4</sup>

If  $B$  is varied while  $R_c$  remains constant, the motional impedance at any given high frequency within the piston range will increase with  $B^2$ . The electrical equivalent of radiation resistance, though small, will increase and with it the ratio, again small, of acoustic power radiated to electrical power input. Thus efficiency varies with  $B^2$ . At the same time the increase of motional impedance while the resistance  $R_c$  remains constant causes  $Q_c$ , the electrical  $Q$ , to decrease inversely with increasing  $B^2$ .

But as the frequency decreases, the motional impedance rises, reaching at  $f_h$  and again at  $f_l$  a maximum value of  $R_{cs}$  which is usually several times the resistance  $R_c$ . Thus at these peaks the motional impedance, which at high frequencies was negligible compared with  $R_c$ , is now the major part of the total impedance. Suppose for simplicity that it comprises all of the speaker impedance. This time when  $B$  is varied and the motional impedance

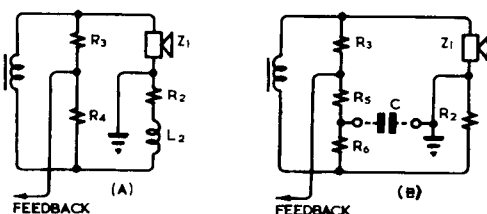


Fig. 14. Methods of eliminating negative output impedance at high frequencies.

<sup>4</sup> This should not be confused with the technique of mass control practiced by politicians and advertising people. In that context, the reactance is usually assumed to result from the equivalent of a compliance, and hence to decrease with signal frequency.

varies as  $B^2$ , then for a given acoustic power output the voltage across  $R_{es}$ , which is virtually the input voltage, will need to increase with increasing  $B$ . Summarizing, for a fixed acoustic power output, an increase of  $B$  will decrease the input voltage required at high frequencies, and increase the input voltage required at the impedance peaks. Also  $Q_c$  will decrease.

With a load impedance much larger than nominal, the criterion of performance of the amplifier becomes, not output power, but the undistorted output voltage on open circuit. This will always be larger than the undistorted output voltage at nominal load; how much larger will depend on the design of the amplifier.

Now if the  $Q_t$  required for a flat frequency response is identical with the  $Q_c$  of the loudspeaker, then if we ignore  $Q_n$ , the generator impedance  $R_g$  must be zero. Thus for a constant acoustic power output the same voltage will be required at the loudspeaker terminals at all frequencies, and all impedances, so that at the frequency  $f_h$  somewhat more maximum acoustic power is available than at higher frequencies.

If the  $Q_t$  required is less than  $Q_c$ ,  $R_g$  will need to be negative, and for constant acoustic power and amplifier output voltage, at the junction of  $R_g$  and  $R_c$  in Fig. 4, will fall at  $f_h$ . But if the  $Q_t$  required is greater than  $Q_c$ ,  $R_g$  will need to be positive, and the amplifier output voltage for constant acoustic power will rise at  $f_h$ . If the ratio of increase of voltage required is greater than the ratio of amplifier undistorted output voltages on open circuit to on-load, it is possible for less maximum acoustic power to be available in the region of  $f_h$  than at other frequencies in the useful band. But since low values of  $Q_c$  are normally associated with high efficiency, this is only likely to occur with high-efficiency, usually high-quality speakers. It should not cause trouble until  $Q_c$  is less than half  $Q_t$ , and even then the maximum acoustic power in most program material is less at frequencies below 100 Hz than around 400 Hz.

Thus there is a paradox that a highly efficient speaker may deliver less power around  $f_h$  than at higher frequencies, while a less efficient speaker delivers more. This will depend on the ratio of  $Q_c$  to  $Q_t$  and of amplifier undistorted output voltage off-load to on-load.

Related to this topic is the flattening of the impedance characteristic which is usually considered to be a good feature of vented boxes. Reference to Fig. 5, and comparison with Fig. 16, shows that, with the simplifying assumption that the resistive losses in the box and vent are negligible, the height of the impedance peak  $R_c + R_{es}$  peaks at  $f_h$  and  $f_l$  and raise the minimum impedance at  $f_b$ . But this is incidental, and the relative heights are of little importance. Thus the idea of tuning the box so that the impedance peaks at  $f_h$  and  $f_l$  are equal, misses the real point. In the impedance curve of a loudspeaker in a box, the most useful information is not the values of the impedances, so long as box and vent damping is not too severe, but the values of the frequencies  $f_h$ ,  $f_b$ , and  $f_l$ . Knowledge of these three frequencies alone enables a box alignment to be checked by Eqs. (105) and (106).

It should be clear that flatness of the impedance characteristic is no indication of flatness of acoustic response. Take as an analogy a coupled pair of tuned circuits. When the output voltage, or more exactly the transfer impedance, is maximally flat, the input impedance has two

peaks. If one parameter is known, say the ratio of primary to secondary  $Q$ , the transfer impedance can be deduced from the input impedance, just as we do for loudspeakers in Eqs. (105) and (106). But a flat input impedance characteristic does not indicate a flat transfer impedance. In a loudspeaker, the impedance characteristic has greater peaks, whose height depends purely on the acoustic damping, though this contributes little to the overall system damping, and thus the overall frequency response.

### XIII. EFFECTIVE REVERBERATION TIME

An objection sometimes made to the use of vented boxes is that the slope of attenuation beyond cutoff, 24 dB per octave, is much steeper than the 12 dB per octave of a speaker on an infinite baffle, and therefore the transient response is worse. In a low-pass filter, the ringing associated with steep attenuation slope is virtually removed by the use of Thompson or critically damped responses. But in high-pass filters such as are considered here, there is always some overshoot with filters of order two or more. To estimate its effect on a listener we use the concept of "effective reverberation time."

Imagine that we have a source of sound in a room which has built up a steady field. The source is then stopped. The sound in the room does not stop immediately, but dies away gradually. The time taken for the sound to decay is called the reverberation time, defined as the time taken for the sound pressure in the room to fall 60 dB from its original value. In small rooms the reverberation time will probably lie between 400 ms for a highly damped room to 1 s or more for a live one.

When the sound passes through two reverberant rooms in cascade, the law of the resulting overall reverberation time is not well established, but calculations on cascaded high-pass filters suggest that rms addition gives at least a guide. In any case it would appear that an added reverberation time of 200–300 ms should not appreciably color the reproduction.

When a transient is applied to a filter and it rings, the effect is perceived by the ear, or brain, as an extension of the transient event in time. Hence the expression "hang-over." To express the effect of the ringing then, an idea is borrowed from architectural acoustics, and the effective reverberation time of a filter is defined as the time taken, after a step function is applied, for the amplitude of the envelope of ringing to fall 60 dB below the amplitude of the original step function.

For the higher order filter functions, with two or more second-order factors, only the most lightly damped factor need be considered. For, by the time the ringing due to the most lightly damped factors is 60 dB down, the ringing due to the more heavily damped factors is negligible. This eases computation greatly.

Actually, at low frequencies the reverberation time defined above will be rather longer than the time the sound is perceived by the listener. To see why, we consult the much abused Fletcher–Munson curves [4, Fig. 12.11].

Suppose, for example, that the original sound is at 100-phon level. This is probably the maximum a system could reproduce, or a listener tolerate. Now at 50 Hz the threshold of hearing is 51 dB above reference level, that is, 49 dB below our arbitrary listening level. At 25 Hz the threshold of hearing is 67 dB above reference

Table IV. Reverberation times for various alignments.

Type of response	B <sub>2</sub>	C <sub>2</sub>	C <sub>2</sub>	B <sub>4</sub>	C <sub>4</sub>	B <sub>6</sub>	C <sub>6</sub>	C <sub>6</sub>	C <sub>6</sub>
Q <sub>t</sub> (for second order alignments)	0.707	1.000	1.414	—	—	—	—	—	—
k (for sixth order alignments)	—	—	—	—	—	1.000	0.600	0.414	0.268
Alignment numbers	—	—	—	5	8	15, 20, 26	17, 22	19, 24	25, 27
Time (in periods of cutoff frequency)	1.63	2.24	3.17	2.87	7.09	4.77	6.79	9.67	14.86
Time for 50 c/s cutoff (mS)	33	45	63	57	142	95	136	193	297

level, that is, only 33 dB below our arbitrary listening level. At 25 Hz, therefore, the effective reverberation time for the listener cannot be greater than the time in which the sound level falls 33 dB, i.e., about half the reverberation time as defined conventionally. Thus at low frequencies in general, the conventional definition based on a 60-dB fall in level yields a reverberation time rather longer than a listener will hear. (This is probably the reason for the observed increase in optimum reverberation time at low frequencies, see [4, Fig. 11.11].)

In a filter which cuts off sharply, the major ringing frequency will be close to the cutoff frequency. Also for a given shape of response curve the reverberation time can be expressed as a certain number of cycles of the cutoff frequency (see Table IV), i.e., the reverberation time increases with decreasing cutoff frequency. On the other hand, below, say, 50 Hz, its effect on the listener will decrease at approximately the same rate. Thus for all filters of a given response curve shape, the figure for 50 Hz should give a rough idea of the maximum reverberation time, as perceived by the listener.

Calculated reverberation times are given in Table IV. The first three alignments are of second order, corresponding to a loudspeaker on an infinite baffle. For these, the values of  $Q_t$  are shown. Note that the reverberation time, though low, doubles as  $Q_t$  increases from 0.707 to 1.414, that is, when the frequency response goes from maximally flat to a 4-dB peak. The times for 50-Hz cutoff are all below 200 ms, except for the last ( $k = 0.268$ ), which is the very steepest.

It thus appears that a properly adjusted vented box, even with amplifier assistance (auxiliary filtering), need cause no perceptible coloration due to ringing. But it is important to emphasize that the adjustment must be correct. Table IV shows that the addition of a 4-dB peak to the response of a speaker on an infinite baffle can double the reverberation time. Being low in the first place it remains tolerable. But in the case of a vented box, particularly with an auxiliary filter, a doubled reverberation time would be more serious. Again, this emphasizes the importance of adequate damping (for correct value of  $Q_t$ ) by the amplifier.

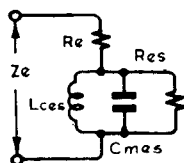


Fig. 15. Simplified equivalent electrical circuit of loudspeaker.

#### XIV. MEASUREMENT OF LOUDSPEAKER PARAMETERS

In earlier sections it was shown how the required response can be obtained from a loudspeaker and box if several parameters are known. The question remains, how are these parameters found?

Properly, this information should be available from the loudspeaker manufacturer. This is particularly important for equipment produced in quantity, where it is important to know not only the mean values but also the tolerances. However, in the absence of published figures, or to check them, the following procedure will provide the information.

Procedures for measuring  $Q$  are given in [2, p. 13], but the method used seems too laborious and inaccurate. The method outlined hereafter can be understood by considering Figs. 15 and 16. Figure 15 is derived from Fig. 4; only this time we omit the vented box and we ignore  $L_e$  and  $R_{sh}$  which take effect at much higher frequencies. Now

$$Q_a = \omega_s C_{mes} R_{es} \quad (91)$$

$$Q_e = \omega_s C_{mes} R_e \quad (92)$$

These quantities, defined earlier in Eqs. (71) and (72) in terms of the acoustic equivalent circuit, are defined here in terms of the electrical equivalent circuit. We define  $r_0$  as the ratio of the impedance at resonance,  $R_{es} + R_e$ , to the dc resistance of the voice coil  $R_e$ . Now we take another arbitrary impedance which is presented at two other frequencies  $f_1$  and  $f_2$  on the flanks of the curve, and we call its ratio to the dc resistance  $r_1$ . Then

$$f_1 f_2 = f_s^2 \quad (93)$$

Physically, this means that the curve is symmetrical on a logarithmic frequency scale. In experimental work it provides a handy check. Now we can find

$$Q_a = [f_s / (f_2 - f_1)] [r_0^2 - r_1^2] / (r_1^2 - 1)]^{1/2} \quad (94)$$

and

$$Q_e = Q_a / (r_0 - 1). \quad (95)$$

If additionally we choose  $r_1$  such that

$$r_1 = \sqrt{r_0} \quad (96)$$

then Eq. (94) is simplified to

$$Q_a = \sqrt{r_0} f_s / (f_2 - f_1). \quad (97)$$

The interesting feature of these expressions is that they involve no approximations, and thus hold for all values of  $Q$ . Furthermore around the value  $\sqrt{r_0}$  the curve has its greatest slope. Thus the frequencies  $f_1$  and  $f_2$  can be

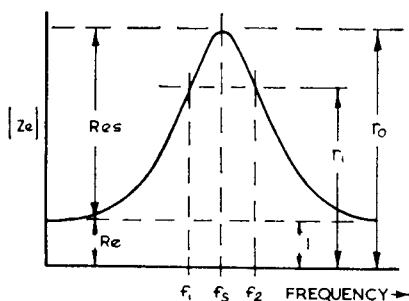


Fig. 16. Typical impedance curve of loudspeaker, modulus of  $Z_e$  in Fig. 15.

found most accurately. This is especially important since the calculation involves a comparatively small difference between large numbers  $f_2 - f_1$ .

Usually  $Q_a$  takes account of the acoustic resistances in the loudspeaker. But if the voice coil has a short-circuited turn by accident or design, e.g., an aluminum former, this will appear in  $Q_a$ , even though its physical nature is similar to  $Q_e$ . (But eddy current losses in the pole piece or front plate appear in  $R_{sh}$ .)

Fig. 17 shows the test circuit.  $V$  is a voltmeter of impedance much higher than the loudspeaker. Throughout the readings, the generator is adjusted so that the reading of  $V$  is constant. The value is not of great importance, but a standard test figure is one volt. The accuracy of this voltmeter is not important so long as it is independent of frequency.  $A$  is an ac ammeter which reads the current into the speaker with the fixed voltage across its terminals. Again, since we are interested only in the shape of the impedance curve, the absolute accuracy of this instrument is not important so long as the meter reading is linear. However, to set the relative current due to  $R_e$ , first we measure  $R_e$  with dc on a Wheatstone bridge, and then a calibrating resistor  $R_c$  of similar value. Connecting  $R_c$  to the test terminals and applying the standard test voltage at say,  $f_s$ , a current value  $I_c$  is found on the ammeter  $A$ . Then the current  $I_e$  which corresponds to  $R_e$  is found by

$$I_e = I_c R_c / R_e. \quad (98)$$

Now the loudspeaker is suspended in air as far from reflecting surfaces as is practical and connected to the test terminals instead of  $R_c$ . The generator is adjusted to the speaker resonant frequency  $f_s$ , indicated by minimum current  $I_o$ . Thus  $r_o$  is found:

$$r_o = I_e / I_o. \quad (99)$$

Now the current  $\sqrt{I_e I_o}$  is found corresponding to the ratio  $\sqrt{r_o}$  and the frequencies either side of resonance, where this current value is read. These are  $f_1$  and  $f_2$  and they should be read to as close an accuracy as the test gear will allow. Eq. (93) provides a check on the

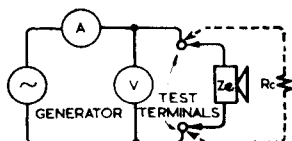


Fig. 17. Test circuit schematic for measurement of loudspeaker parameters.

method, and Eqs. (97) and (95) give  $Q_a$  and  $Q_e$ .

The next problem is to find the value of  $V_{as}$ , the volume of air equivalent to the loudspeaker compliance. For this, the loudspeaker is placed in a totally enclosed unlined box whose internal volume  $V_b$  is known, remembering that allowance must be made for bracing and the volume displaced by the speaker. It is important that this box be free of air leaks. If these occur we will read part of the curve of Fig. 5, around  $f_h$ . Thus care should be taken, not only in the construction of the box and in the mounting of the speaker, but also in the way the speaker leads are taken through the walls of the box. Solid terminals are preferred.

Another precaution may be necessary. In Figs. 15 and 16, from which we derived Eqs. (93), (94), (95), and (97), we assumed that the effect of the inductance  $L_e$  is negligible. In fact,  $L_e$  interacts with the parallel combination of  $L_{ces}$  and  $C_{mes}$  to produce a series resonance at  $f_n$  in Fig. 5, where the nominal impedance is measured. If this frequency, usually 400–600 Hz, is well above the speaker resonance  $f_s$ , so that there is little disturbance of the curve at  $f_2$  of Fig. 16, the accuracy of the measurements will be unaffected. But if  $f_s$  is above 150 Hz, which can occur with small speakers and becomes even more likely when the speaker is placed in the box for the last test, the likelihood of inaccurate results increases.

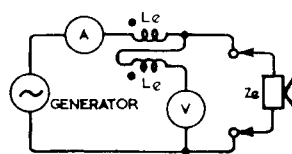


Fig. 18. Modification of Fig. 17 to cancel effect of loudspeaker inductance  $L_e$ .

This could be avoided by connecting in the circuit of Fig. 18 a bifilar inductance whose value  $L_e$  in each half is equal to the inductance of the voice coil. It is preferable, and not difficult, to wind this with an air core. In measuring  $L_e$  of the loudspeaker, it is important to measure it at a frequency well away from  $f_n$ , say, 10 kHz. Also it is important to measure it as an inductance in parallel with a resistance ( $D$  or  $\tan \delta$  scale, not the  $Q$  scale of a bridge), for the  $Q$  of the inductance at 10 kHz is usually of the order of one which can lead to serious error if the measurement is made as an inductance in series with a resistance. With a high-impedance voltmeter  $V$ , error due to series resistance of the inductor should be negligible.

If the new resonant frequency in the closed box  $f_{sc}$  is found, the ratio of volume is usually given as

$$V_{as}/V_b = (f_{sc}/f_{sa})^2 - 1 \quad (100)$$

where  $f_{sa}$  is the resonant frequency of the speaker in air which we previously called  $f_s$ . However, this expression ignores the change in the acoustic mass  $M_{as}$  of 1.05 to 1.25 times which results from placing the speaker in the box. A more accurate method is to repeat the previous procedures for finding  $Q_e$ . Then if we call  $Q_{ea}$  and  $Q_{eo}$  the values of  $Q_e$  measured in air and in the closed box, respectively, then

$$V_{as}/V_b = [(f_{sc} Q_{ec}) / (f_{sa} Q_{ea})] - 1. \quad (101)$$

Also the ratio of the acoustic masses in air and in the closed box

$$M_{asa}/M_{asb} = f_{sa}Q_{ca}/f_{sa}Q_{ce} \quad (102)$$

should lie between 0.8 and 0.95.

With  $V_b$  known,  $V_{as}$  can be calculated. The size of  $V_b$  is not critical, but should not be too large, otherwise the ratio  $f_{sa}/f_{sb}$  becomes close to unity, and the accuracy of the  $V_{as}/V_b$  calculation falls. This can be seen from Eq. (100). Finally the values of  $f_{sa}$  and  $Q_{ca}$  are adjusted to take account of the change in  $M_{as}$  when the speaker is placed in the box. Thus

$$f_{sb} = f_{sa}(M_{asa}/M_{asb})^{1/2} \quad (103)$$

$$Q_{cb} = Q_{sa}/(M_{asa}/M_{asb})^{1/2}. \quad (104)$$

Thus the efficiency  $\eta_{pb}$  can be calculated from Eq. (77). This gives the result, rather surprising at first sight, that the electroacoustic conversion efficiency of a loudspeaker in the piston range can be calculated from electrical measurements alone.

The following alternative method is useful, particularly when the loudspeaker has to be placed in a box whose size is already determined or as a final check on a previously calculated box, or again if it becomes too difficult to seal the loudspeaker in the test box.<sup>5</sup>

First the vent, if adjustable, is made to resonate with the box somewhere near the speaker resonant frequency, but this is not very important. Then the three frequencies  $f_i$ ,  $f_b$ , and  $f_h$  of Fig. 5 are found as accurately as possible. Special care is needed in reading  $f_b$  as the curve has a flat bottom.

From these readings we find  $f_{sb}$ , the resonant frequency of the speaker when mounted in the box,

$$f_{sb} = f_h f_i / f_b \quad (105)$$

and the compliance ratio  $C_{as}/C_{ab}$ , i.e.,

$$V_{as}/V_b = (f_h^2 - f_b^2)(f_b^2 - f_i^2)/f_h^2 f_i^2. \quad (106)$$

With the speaker resonant frequency in air  $f_{sa}$  already known and  $f_{sb}$  known from Eq. (105), we find the mass ratio  $M_{asa}/M_{asb}$  from Eq. (103), and then  $Q_{cb}$  from Eq. (104).  $Q_a$  is adjusted to  $Q_{ab}$  in a similar manner. By reference to Table I and Fig. 7, a suitable alignment can be found, thus setting the final values of  $f_b$  and  $Q_t$ . Note that  $Q_t$  is due to the parallel combination of 1)  $Q_{ab}$  and 2)  $Q_{cb}$  modified by the amplifier.

To estimate the value of  $Q_b$ , the "Q of the box and vent circuit," we measure  $I_b$ , the current through the speaker at  $f_b$ , with the input voltage held constant as before. Then

$$Q_b = (\omega_b/\omega_s)(C_{ab}/C_{as})[(1/Q_c) + (1/Q_a)][(I_b - I_o)/(I_c - I_b)]. \quad (107)$$

<sup>5</sup> Experience gained since the writing of this paper shows that accurate results are more easily obtained with this second method. Using a vented box is especially preferred if the speaker being measured has a low resonant frequency and if the testing box is fairly small. In such cases, small leaks in the "totally enclosed" box or around the loudspeaker pad ring can produce a virtual vent which produces the familiar twin peaks of loudspeaker impedance. But if the lower peak is below the limit of measurement, say, below 10 or 15 Hz, it could easily happen that the remaining upper peak would be taken as the single peak of a closed-box system with dire results.

Note that, because the difference between  $I_c$  and  $I_b$  will be small, the readings must be taken carefully.

Comparing Eq. (107) with Eq. (89), it can be seen that

$$(A_b)_{max} = 1/\{1 + [Q_a Q_c / Q_t (Q_a + Q_c)][(I_c - I_b)/(I_b - I_o)]\}. \quad (108)$$

This greatly simplifies the estimation of  $(A_b)_{max}$ .

A worked example of this method is given in the Appendix.<sup>6</sup>

## XV. EXPERIMENTAL WORK

When the work was started from which this paper derived, it was necessary first to find the parameters for a number of loudspeakers. To date about fifty have been measured. In the case of one speaker, the effect of a number of modifications was observed; in the rest, usually one and occasionally two or three samples have been checked. The results obtained give confidence in the method. For example, from the readings and knowing other parameters, it is possible to calculate the flux density, and the values obtained give good correlation with readings on a flux meter. Changes of parameters during production can also be detected.

Some generalizations from the results have been mentioned earlier. For example, it was found that  $Q_a$  varies between about 3 and 10, which is high compared with the  $Q_t$  values of 0.2 to 0.6 required in Table I. Thus it was apparent that acoustic resistance usually has little effect on the damping of a speaker in a well-designed system. Values of  $Q_c$  varied from 0.2 to 0.5 in the case of high-quality speakers, through 0.5 to 1.0 in the better commercial grades of speakers, to 2 and even 3 in the case of some low-priced speakers.

Similarly efficiencies, for radiation from one side of an infinite baffle, ranged from -24 dB (0.4%) for low-priced speakers through -20 dB (1%) for medium-grade to -14 dB (4%) for high-quality speakers.

However, one must resist the tempting generalization that it is possible to rate the overall quality of a speaker by its  $Q_c$  or even its efficiency. For example, if efficiency is made higher and  $Q_c$  lower by reducing the cone mass  $M_{ms}$ , trouble with "break up" may result at middle frequencies. In fact while the best 8-in speaker tested had a  $Q_c$  of 0.33, there was one sample with good clean response at high frequencies with a high  $Q_c$  of 1.7 and another with  $Q_c$  below 1 which was less acceptable. It must be remembered that these readings, and the paper in general, are concerned only with low-frequency performance.

As a result of the design theory, a number of boxes have been made. In the absence of reliable measurements of sound pressure, all that can be said is that they gave a good improvement in clean low-frequency response, and that the cutoff frequencies are near the predicted values. Some particularly gratifying results have been obtained

<sup>6</sup> Experimental work, using the above method indicates that in practical boxes  $Q_b$  is often of the order of 10. This difference from the calculated values of 30 or more may be due to frictional losses in the timber. It is shown in Section XI that when  $Q_b$  is 10, the frequency response error is still only 1 to 2 dB. However, if there are sufficient air leaks, or if the cavity damping is excessive, as when the box is completely stuffed with underfelt,  $Q_b$  can fall below 5.

with 5-in speakers in modest boxes with response down to 80 Hz.

## XVI. CONCLUSION

The work described herein was begun as an advanced development project in an attempt to obtain good low-frequency response from loudspeakers in small boxes. Unfortunately, no "revolutionary concept" was uncovered that offers something for nothing. On the other hand, it has provided a reasonably precise method of design that was previously lacking.

In general, a system with good flat response down to a predictable cutoff frequency can be designed, if the necessary parameters  $Q_c$  (and  $Q_a$ ),  $V_{as}$ , and  $f_s$  are known for the loudspeaker. The box volume is closely proportional to the inverse square of cutoff frequency, which can be varied over a wide range. The output impedance  $R_o$  of the amplifier has a large effect in controlling the response, especially at  $f_n$ , the higher frequency of maximum impedance. Whether  $R_o$  needs to be positive, zero, or negative depends on the type of alignment and the  $Q$  parameters of the speaker. On the evidence available, acoustic resistance damping of the vent has no advantage, and is wasteful of box volume or bandwidth.

The advantages accruing from a predictable design include the possibility of optimum design of "rumble" filters. At frequencies below cutoff where negligible acoustic output is produced, these relieve the amplifier and loudspeaker of high signal amplitudes and thus minimize an annoying source of intermodulation distortion. Carried a step further, the use of auxiliary electrical filters makes it possible to trade box volume for low-frequency power capability of the amplifier.

Another way of reducing box volume is to increase the mass of the loudspeaker cone. But since this also reduces efficiency, it may be considered as a further example of trading amplifier size for box size, only this time the amplifier must deliver increased power over the whole audio spectrum. Again, the box volume may be reduced if a smaller diameter loudspeaker is used. The danger here is that the speaker excursion increases, but it is a good solution if the speaker is capable of a long linear excursion, or if the power output and/or low-frequency response is restricted.

The size of the magnet, or more precisely the flux density  $B$ , has a great influence on performance. Both efficiency, hence acoustic output, and  $Q_c$  vary with  $B^2$ ; so it is clear that the saving of pennies on a smaller magnet can be poor economy.

The parameters needed for vented-box design can be measured with normal electrical measuring equipment together with a test box of known net internal volume. Nevertheless it is suggested to loudspeaker manufacturers that it is in their interest, as well as the user's, to publish typical values of  $Q_c$ ,  $Q_a$ ,  $V_{as}$ , and  $x_{max}$ , as well as  $f_s$ . These parameters are more useful to the system designer than, for example, flux density or total flux. Their publication would help ensure that the manufacturer's product is used to the best possible advantage.

The totally enclosed box has been mentioned only in passing, since it is well covered in [2]. But it should be noted that if a totally enclosed box is chosen with the same volume as that of alignment no. 5, the cutoff frequency is 1.55 times higher. With smaller boxes, the advantage

decreases, though with practical sizes it is still appreciable. With larger totally enclosed boxes, the cutoff frequency can never fall below  $f_s$ , while the Chebyshev vented box alignments can extend the response considerably below  $f_s$ .

The greatest advantage of a vented box over an infinite baffle is the reduction of loudspeaker excursion, permitting higher power output or lower distortion. To this advantage, the present paper adds, it is hoped, a greater flexibility in design. The only apparent disadvantage of a vented box is in the transient response, but in fact the ringing is only perceptible with a misadjusted alignment. With proper adjustment, the effective reverberation time, though longer than that of a properly adjusted infinite baffle, is not long enough to appreciably color the sound in the listening room.

Finally, it is emphasized again that the acoustic response is due to the combination of speaker plus box plus amplifier as an integrated whole.

## APPENDIX: WORKED EXAMPLE

This refers to a purely imaginary speaker, the readings being chosen to simplify the calculations. However, the readings would be typical of a medium-quality 8-in speaker.

### Measurement of Speaker Parameters

$Q_a$ ,  $Q_c$ ,  $V_{as}$ , and  $f_s$

With a Wheatstone bridge we find

dc resistance of speaker  $R_c = 4.00$  ohms

dc resistance of calibrating resistor  $R_e = 5.00$  ohms.

Now we place  $R_e$  in the test circuit of Fig. 17 and find that when  $V$  reads 1 volt,

$$I_c = 180 \text{ mA.}$$

Now

$$I_c R_c = 0.180 \times 5.00 = 0.900.$$

Since this is 10% below the observed reading of 1 volt, one or both of the meters is inaccurate, but this is unimportant so long as their readings are constant with frequency and the reading of ammeter  $A$  is linear.

Then from Eq. (98),

$$I_c = I_c R_c / R_c = (0.180 \times 5.00) / 4.00 = 225 \text{ mA.}$$

We now suspend the loudspeaker in air as far from reflecting surfaces as possible and read the minimum current  $I_0$  which is 25 mA at 55.0 Hz ( $f_{sa}$ , the speaker resonant frequency in air).

Then from Eq. (99),

$$r_0 = I_c / I_0 = 225 / 25 = 9$$

$$\sqrt{r_0} = \sqrt{9} = 3$$

$$\sqrt{(I_0 I_c)} = \sqrt{(225 \times 25)} = 75 \text{ mA.}$$

With the voltmeter  $V$  reading a constant 1 volt, the ammeter  $A$  reads 75 mA at 44.0 and 68.75 Hz.

First we use this reading to check  $f_{sa} = \sqrt{(44.0 \times 68.75)}$  from Eq. (93) = 55.0 Hz as before. Then from Eq. (97),

$$Q_a = f_a \sqrt{r_0} / (f_2 - f_1) = (55 \times 3) / (68.75 - 44) = 6.67$$

and from Eq. (95),

$$Q_e = Q_a/(r_o - 1) = 6.67/(9 - 1) = 0.833.$$

The speaker is now placed in a vented box whose net volume is 1000 in<sup>3</sup> and we read the frequencies defined in Fig. 5,

$$f_h = 100 \text{ Hz}; \quad f_b = 60 \text{ Hz}; \quad f_l = 30 \text{ Hz}.$$

Then from Eq. (105),

$$f_{sb} = f_h f_l / f_b = (100 \times 30) / 60 = 50 \text{ Hz}$$

and from Eq. (106),

$$V_{as}/V_b = (f_h^2 - f_b^2)(f_b^2 - f_l^2)/f_h^2 f_l^2.$$

Computation is easier if we rewrite Eq. (106) as

$$V_{as}/V_b = (f_h + f_b)(f_h - f_b)(f_b + f_l)(f_b - f_l)/f_h^2 f_l^2$$

i.e.,

$$\begin{aligned} V_{as}/V_b &= (100 + 60)(100 - 60)(60 + 30)(60 - 30)/ \\ &\quad 100^2 \times 30^2 \\ &= (160 \times 40 \times 90 \times 30)/(100 \times 30 \times 100 \times 30) \\ &= 1.92 \end{aligned}$$

i.e.,

$$V_{as} = 1.92 \times 1000 = 1920 \text{ in}^3.$$

In the vented box, the speaker resonant frequency has dropped  $f_{sb}/f_{sa} = 50/55 = 0.909$  times. Thus from Eq. (103),

$$M_{asa}/M_{asb} = (0.909)^2 = 0.826$$

and from Eq. (104),

$$Q_{ab} = 6.67/0.909 = 7.33$$

while

$$Q_{cb} = 0.833/0.909 = 0.917.$$

At  $f_b$  the current  $I_b$  was read as 220 mA. Then from Eq. (107), the  $Q$  of the box plus vent

$$\begin{aligned} Q_b &= (f_b/f_s)(C_{ab}/C_{as})[(Q_a + Q_e)/Q_a Q_e] \\ &\quad [(I_b - I_o)/(I_e - I_o)] \\ &= [60 \times (7.333 + 0.917) \times (220 - 25)] / \\ &\quad [50 \times 1.92 \times 7.33 \times 0.917 \times (225 - 220)] \\ &= (60 \times 8.25 \times 195) / (50 \times 1.92 \times 7.33 \times 0.917 \times 5) \\ &= 29.9. \end{aligned}$$

From Eq. (108) the maximum box loss in the quasi-Butterworth alignment described below, where  $Q_t = 0.347$ , is

$$\begin{aligned} (A_b)_{max} &= 1/\{1 + [Q_a Q_e / Q_t(Q_a + Q_e)] \\ &\quad [(I_e - I_b)/(I_b - I_o)]\} \\ &= 1/\{1 + (7.33 \times 0.917 \times 5) / \\ &\quad (0.347 \times 8.25 \times 195)\} \\ &= 1/1.060 \end{aligned}$$

which is equivalent to 0.5 dB.

**Efficiency  $\eta$  from Eq. (77)**

$$\begin{aligned} \eta_{ob} &= 8.0 \times 10^{-12} f_s^3 V_{as} / Q_e \\ &= (8.0 \times 50^3 \times 1920) / (10^{12} \times 0.917) \\ &= 2.09 \times 10^{-3} \end{aligned}$$

which is equivalent to -26.6 dB in a box, or -23.6 dB on an infinite baffle (i.e., a true infinite baffle, not a totally enclosed medium-sized box which gives the same efficiency as a vented box), or -20.6 dB on a true infinite baffle, taking into account radiation from both front and back.

Thus if the speaker is mounted in a box and fed a 5-watt amplifier, the acoustic power output will

$$W_{ao} = \eta_{ob} W_{ei} = 5 \times 2.09 \times 10^{-3} = 0.0104 \text{ Watt}$$

If we assume a listening room of  $16 \times 12\frac{1}{2} \times 10 = 2000 \text{ ft}^3$ , then from [4, p. 418, Fig. 11.12] an acoustic power of 0.003 watt provides +80-dB intensity level. Our output is 10.4/3 times, i.e., 5.4 dB greater than this; therefore the system is capable of a peak +85-dB intensity level.

**Peak Excursion  $x_{pk}$**

We assume an alignment where the box is tuned to the same frequency as the loudspeaker, i.e., 50 Hz. This is typical of Butterworth alignments. Then the fixed part of the expression for  $x_{pk}$  in Eq. (84) is

$$(1.31 \times 10^5 \times \sqrt{W_{ao}}) / f_b^2 S_d.$$

Now if the effective piston diameter is 7 in, i.e.,

$$S_d = \pi \times 3.5^2 = 38.5 \text{ in}^2$$

then the expression becomes

$$1.31 \times 10^5 \times \sqrt{0.104} / (50^2 \times 38.5) = 0.139 \text{ in}.$$

Now the maximum value of the frequency-sensitive expression for a vented box in the useful band (above  $f_b$ ) in Fig. 10 is approximately one quarter. Thus

$$x_{pk} = 0.139/4 = \pm 0.035 \text{ in}$$

compared with  $\pm 0.098 \text{ in}$  in a totally enclosed box (infinite baffle).

**Box Design**

First suppose we wish to obtain the best results with the original 1000-in<sup>3</sup> box. Allowing 10% for the bracing and volume displaced by the speaker, the optimum inside dimensions would be  $\sqrt[3]{1100 \times (0.8, 1.0, 1.25)} \text{ in}$ , i.e.,  $8.28 \times 10.33 \times 12.9 \text{ in}$ , say  $8\frac{1}{4} \times 10\frac{1}{4} \times 13 \text{ in}$ . This would need to be checked in case the original assumption of 10% was incorrect. Assuming that the dimensions are

Table V. Computation of three Butterworth alignments for imaginary speaker.

Type of alignment	QB <sub>3</sub>	B <sub>4</sub>	B <sub>5</sub>	B <sub>6</sub> (i)
$C_{as}/C_{ab}$	1.92	1.414	1.000	2.732
$V_b$ (cubic inches)	1000	1358	1920	704
Box {	Height (in.)	13	14	16
	Width (in.)	10 $\frac{1}{4}$	11 $\frac{1}{2}$	13
	Depth "d" (in.)	8 $\frac{1}{4}$	9	10
Cutoff frequency $f_3$ (c/s)	58.5	50	50	50
Box frequency $f_b$ (c/s)	54.7	50	50	50
$L_v/S_v$ (in. <sup>-1</sup> )	1.56	1.37	0.97	2.65
$S_v$ (in. <sup>2</sup> )	7.69	10.07	16.25	4.50
Vent height "1" (in.)	$\frac{3}{4}$	$\frac{7}{8}$	1 $\frac{1}{4}$	$\frac{1}{2}$
$Q_t$	.347	.383	.447	.299
$(Q_s)_{total}$	.364	.404	.476	.312
$R_g/R_e$	-.600	-.560	-.481	-.660

then in a box similar to Fig. 9, the width of will be  $10\frac{1}{4}$  in. The length of the tunnel will be  $9\frac{3}{4}$  in, together with two thicknesses of timber (say  $\frac{1}{2}$  in each) plus a  $\frac{1}{2}$ -in square stiffener on the top rear edge of the shelf, giving a total tunnel length of  $9\frac{3}{4}$  in.

The simplest alignment for  $C_{as}/C_{ab} = 1.92$  is a third-order quasi-Butterworth between alignments no. 4 and 5. From Fig. 7 (b),

$$f_3/f_s = 1.17, \text{ thus } f_3 = 50 \times 1.17 = 58.5 \text{ Hz}$$

$$f_3/f_b = 1.07, \text{ thus } f_b = 58.5/1.07 = 54.7 \text{ Hz.}$$

Thus

$$\omega_b^2 = 1.18 \times 10^5$$

and for the tunnel, from Eq. (61),

$$\begin{aligned} (L_v/S_v)_{\text{required}} &= 1.84 \times 10^8 / \omega_b^2 V_b \\ &= 1.84 \times 10^8 / 1.18 \times 10^5 \times 10^3 \\ &= 1.56 \text{ in}^{-1}. \end{aligned}$$

Now if the tunnel height  $l = \frac{3}{4}$  in, then area

$$S_v = 10\frac{1}{4} \times \frac{3}{4} = 7.69 \text{ in}^2$$

and

$$\begin{aligned} (L_v/S_v)_{\text{end}} &= 0.958 / \sqrt{S_v} \\ &= 0.958 / \sqrt{7.69} \\ &= 0.34 \text{ in}^{-1} \end{aligned}$$

$$\begin{aligned} (L_v/S_v)_{\text{tunnel}} &= 9.75/7.69 \\ &= 1.27 \text{ in}^{-1}. \end{aligned}$$

Thus

$$(L_v/S_v)_{\text{available}} = 1.61 \text{ in}^{-1}$$

which is about as close as can be obtained with the tolerances on the small dimension ( $\frac{3}{4}$  in) of  $l$ .

### Amplifier Output Impedance $R_g$

Now by interpolation,

$$Q_t = 0.347$$

and since  $Q_{ab} = 7.33$ ,  $Q_{cb} = 0.917$ , and from Eq. (70),

$$1/Q_t = 1/Q_a + 1/Q_c(1 + R_g/R_e).$$

Thus

$$1/0.347 = 1/7.33 + 1/0.917(1 + R_g/R_e).$$

Hence

$$R_g/R_e = -0.60.$$

Notes

1)  $(L_v/S_v)_{\text{end}}$  is small compared with  $(L_v/S_v)_{\text{tunnel}}$

and since the vent area is already small compared with the piston area, a simple hole in the front panel would be quite impractical as a vent. Its area would need to be about  $1 \text{ in}^2$ .

2) The dimension  $l$  ( $\frac{3}{4}$  in) is fairly critical.

3)  $Q_a$  has little effect on  $Q_t$ . The negative impedance required is fairly high but quite practical.

For comparison three Butterworth alignments have also been computed for this imaginary speaker so that the effect of amplifier filtering can be assessed (Table V). All three have cutoff frequencies of 50 Hz. But while  $B_4$  has no filtering,  $B_5$  has a simple CR filter which is  $-3 \text{ dB}$  at 50 Hz ( $CR = 3180 \mu\text{s}$ ), and  $B_6$  has a peak 6 dB high at 53.5 Hz before it falls off at the rate of 12 dB per octave ( $y = -1.732$ ,  $f_{aux} = 50 \text{ Hz}$ ).

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